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Question Paper Code : 91360

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2025.

Third Semester

Electrical and Electronics Engineering

MA 3303 — PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State Baye's theorem.
2. The number of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that the computer will function for a month without a breakdown.
3. If two random variables X and Y have probability density function given by $f(x, y) = kxye^{-(2x+y)}$, for $x > 0, y > 0$. Find the value of k .
4. State central limit theorem for independence and identically distributed random variables.
5. Show that the function $f(z) = z\bar{z}$ is nowhere analytic.
6. Find the fixed points of the mappings $w = \frac{1-z}{1+z}$.
7. State Cauchy's integral theorem.
8. Find the poles of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ and the residue at any one pole.
9. Solve $(D^2 + 6D + 5)y = 16e^{3x}$.
10. Define Legendre's linear equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the density function of a continuous random variable X is given

$$\text{by } f(x) = \begin{cases} ax, & 0 \leq x \leq 1 \\ a, & 1 \leq x \leq 2 \\ 3a - ax, & 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

(1) Find the value of 'a'.

(2) Find the distribution function $F(x)$. (8)

- (ii) The density function of a random variable X is given by

$$f(x) = \frac{3}{4}x(2-x).$$

Find (1) Mean (2) Variance (3) r^{th} moment. (8)

Or

- (b) (i) Out of 800 families with 4 children each, how many families would be expected to have

(1) 2 boys and 2 girls

(2) atleast 1 boy

(3) atmost 2 girls. (8)

- (ii) The time in hours required to repair a machine is exponentially distributed with $\lambda = \frac{1}{2}$.

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the probability that a repair takes atleast 10 hour given that its duration exceeds 9 hour? (8)

12. (a) (i) The joint probability mass function of (X, Y) is given by $f(x, y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Determine all the marginal and conditional probability distributions. (8)

- (ii) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denote the number of white balls drawn and Y denote the number of red balls drawn, find the joint probability distribution of (X, Y) . (8)

Or

- (b) (i) Two random variables X and Y have the joint density

$$f(x, y) = \begin{cases} 2 - x - y, & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find $\text{cov}(X, Y)$. (8)

- (ii) If X and Y are independent RVS with pdf $f_X(x) = e^{-x}, x \geq 0$ and $f_Y(y) = e^{-y}, y \geq 0$, find the pdf of $U = \frac{X}{X+Y}$ and $Y = X + Y$. (8)

13. (a) (i) If $f(z)$ is a regular function of z , prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2. \quad (8)$$

- (ii) Show that a bilinear transformation maps the totality of circles and straight lines in the z -plane onto the totality of circles and straight lines in the w -plane. (8)

Or

- (b) (i) If $f(z) = u + iv$, is analytic function of z , and if

$$u = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}, \text{ determine } v. \quad (8)$$

- (ii) Construct the bilinear map which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. (8)

14. (a) (i) Evaluate $\int_C \frac{dz}{(z-1)(z-2)(z-3)}$ where C is $|z| = 4$. (8)

- (ii) Expand $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in a Lauernt's series of $|z| < 2$. (8)

Or

- (b) (i) Evaluate $\int_C \frac{dz}{(z^2 + 4)^2}$ where C is the circle $|z - i| = 2$. (8)

- (ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ using contour integration, where $a > b > 0$. (8)

15. (a) (i) Solve the equation $(D^2 - 5D + 6)y = e^x \cos 2x$. (8)

(ii) Solve the simultaneous equation $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$. (8)

Or

(b) (i) Solve $(2x + 3)^2 y'' - (2x + 3)y' - 12y = 6x$. (8)

(ii) Solve the equation $\frac{d^2 y}{dx^2} + y = \sec x$ by the method of variation of parameters. (8)